

Algorithm for Finding a Hamilton Circuit of a Cyclic Graph

A Hamilton circuit is a continuous closed path, through some of the edges of a cyclic graph, that traverses every node just once. Hamilton's own examples were the dodecahedron andicosahedron, which can be represented as planar nets; the circuit being marked, as follows :

(linear programming and graph isomorphism)

~~When~~ a Hamilton circuit (HC) ^{when they} can be found, can be very useful in solving network problems & be the present instance, the impetus for an efficient algorithm to find an HC came from the systematics of chemical graphs, i.e., of the molecular structure of organic compounds. On hindsight, problem was to search for and verify the ^{the} smallest, trihedral polyhedron locking an HC (~~which~~ Tait conjectured to be nonexistent; but Tutte displayed an example of order 46).

of ways of accessing every node of the cut graph.

To form a Hamilton circuit, two cut graphs must be united, edge by edge, which satisfy ~~the~~ the requirement that their lane sets are mutually compatible.

~~to do~~ At each stage of the tree, the lane set is ~~the~~ first expanded, each node bringing pendent, cut edges, either of which is at this point equally available as a lane. Then the cut edges are scanned for ~~identities equimune with one another; such~~ identities which ^{thereby} edges can now be joined and removed from the set of cut edges. At this time, the lane-set is diminished with respect to items involving the re-united edge. If the lane-set is ~~still~~ void

^{The} ~~Polytopes~~ discussion will be illustrated mainly with such polyhedra regarded as undirected graphs or 3-nets. In a fairly obvious way, however, the argument can be extended to cyclic graphs generally.

These graphs are ~~not~~ trivalent at each node and everywhere at least 3-connected. That is, the 3-net cannot be separated by fewer than 3 cuts. It is also planar, i.e., can be represented on the plane (Shlegel diagrams) or on the ~~the~~ 2-sphere (^{convex} polyhedra) with no crossing edges. (number of nodes)

n stands for the order^a of a graph or cutgraph

q for the ~~list~~ of cut edges

p for the connectedness in a special sense:
the ^{list} number of cuts needed to extract a cutgraph larger than a single node or node-pair. Evidently $3 \leq p \leq 5$. Recall also Euler's result that a polyhedron must have at least one face, $3 \leq n \leq 5$.

g_i stands for a graph drawn in Figure .

$[N]$ stands for the incidence matrix (combinatorial)

of the graph, showing which nodes adjoint which

$[E]$ is the corresponding matrix for edges.

The only problem attacked here is a combinatorial one, that can rapidly make a brute force approach unfeasible by a fast computer. Consider two approaches.

(a) $n!$ Apply the symmetric group S_n to the labels of the nodes. Test each permutation for the strict adjacency of the nodes. This approach is prohibitive well before $n \geq 14$.

(b) 2^n A well-known method (BFS) applies a binary search. From n , a binary choice is made between two of its adjacent nodes, and this process iterated until a ~~HC is found~~, ^{or} the path closes. If this path encompasses n nodes (hence n choices) then a HC has been found. Then the ~~alternative choice is made~~ of the path is retraced to the last option point and the alternative choice selected. All possible paths can be described with n bits, hence the ~~max~~ upper limit is 2^n choices. However, many such trees will be truncated quite early. In practice, about $2^n/n^2$ choices have sufficed to find one HC where it exists.

* choosing

the third node will tree paths that must be reflected through the others. Some reflections will already be included

and quite verifiable

This method is feasible through $n \geq 20$,
~~and trivial through $n \leq 12$.~~ It has been used
for a complete testing of the HCs of most of the
cyclic graphs relevant to organic chemistry, i.e.,
all trivalent graphs, $n \leq 12$, and many 3-nets
 $n \leq 20$.

For recent applications at $n > 12$ binary
search is still troublesome. Furthermore it gives up
little insight into the conditions for the existence
of an HC in some larger 3-net, say $n > 30$. Note
that Tutte's example is n_{46} , although one
example has been found (independently also by
and by)
at n_{38} .

A tree-building method. The net can be searched
(or built) ~~in the fashion of~~ as a tree, starting from an
arbitrary node. Each branch is a cut edge, obtained
by placing a node ^{and its pendant edges.} ~~near~~ of the existing cutedges,
~~This process is governed by~~
according to $[N]$ and $[E]$.

We now define a lane-set and the lane-set list.
A lane-set is ~~a pair, or set of pairs~~ one or more
pairs of cut edges which ~~might be~~ are possibly
available for Hamilton circuits; if ~~just~~ it is the set